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Optimal Catch Capacity and Fishing Effort in Deterministic and Stochastic Fishery Models*

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ABSTRACT

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This paper is a survey of fisheries economics, aimed mainly at fisheries biologists. The paper begins by reviewing the static theory, which established two major results. (i) Free access leads to over-exploitation, and (ii) the optimal rate of exploitation is less than the maximum sustainable yield rate. The latter could be regarded as an antithesis to the biological doctrine that fish stocks should be managed to give maximum sustainable yield (*MSY*).

Dynamic theory, which is considered next, showed that the optimal rate of exploitation could be either less or greater than the *MSY* rate. In particular, a higher discount rate was shown to imply a higher rate of exploitation. This, however, ignores the role of capital invested in the harvesting sector. Once it is recognized that a higher discount rate implies a higher required rate of return on capital, the impact of the discount rate on the optimal rate of exploitation becomes ambiguous. The paper examines the impact of the discount rate with and without stock-dependent harvesting costs. This leads on to the question of how the risk of extinction under free access depends on the sensitivity of unit harvesting costs, or catch per unit of effort, to the size of the exploited stock.

Finally, stochastic fishery models are briefly considered. The main purpose of this part of the paper is to demonstrate that deterministic fishery models may give poor guidance for managing the stochastic fisheries of the real world, even if risk neutrality and constant prices are assumed. To demonstrate this as clearly as possible, the unit cost of harvesting is assumed to be constant, implying that the optimal rate of exploitation is constant in a deterministic model. First we consider the simple case of time-invariant stochastic catch quotas (no population dynamics), and demonstrate how optimal catch capacity depends on the cost of investing in the necessary equipment. Then we consider population dynamics, where expected future catch quotas depend on how much is being taken presently. Optimal catch capacity depends on the cost of investment in this case as well, but the derivation of optimal harvesting and investment policies becomes more complicated.

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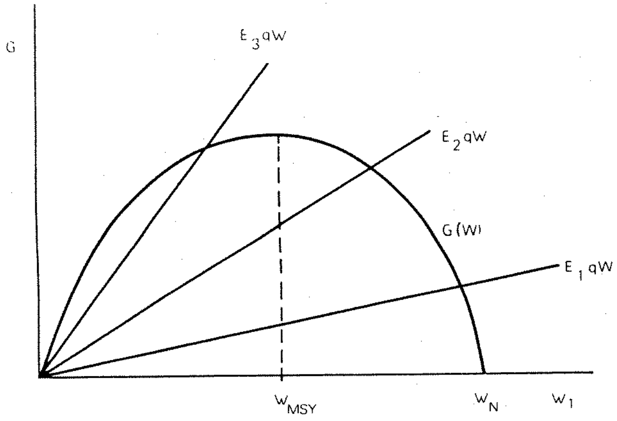

Fig. 1. The surplus growth function (G) of a fish stock (W), and catch as a function of stock, for given levels of effort (E), $E_1 < E_2 < E_3$.

surplus growth by changes in the age-composition of the stock (the Beverton—Holt model), changes in the recruitment of young fish as the spawning stock is diminished (the Ricker model), or even more complex relationships. Common to them all, however, is that surplus growth requires a reduction of the fish stock below its natural equilibrium level. The celebrated model of Schaefer is in fact as simple as the one shown in Fig. 1.

Next we need to relate the landings of fish to the size of the fish stock and its surplus growth in order to approach questions related to economics and management. We shall begin by doing this in the simplest possible manner. Suppose that the catch of fish is proportional to the amount of fish available (W), for any given level of fishing "effort". The straight lines EqW in Fig. 1 illustrate this relationship. Fishing effort is some measure of the activity directed at catching fish, e.g. the number of boats employed per unit of time, adjusted for differences in efficiency. The definition and measurement of fishing effort is a Pandora's Box of problems, which we shall keep solidly closed this time. Some of these problems are discussed in Hannesson (1983b).

The intersections between the lines EqW in Fig. 1 and the surplus growth curve $G(W)$ constitute biological equilibria for an exploited stock, as the amount caught per unit of time is then equal to the surplus growth per unit of time. In Fig. 2, these equilibria are shown as a relationship between the equilibrium catch per unit of time (Y) and fishing effort (E). The top of the curve $Y(E)$ shows the maximum sustainable yield (MSY), that is, the highest yield obtainable in the long term after the fish stock has attained the biological equilibrium associated with the prevailing level of fishing effort. To the maximum sustainable yield there corresponds a certain equilibrium level of the fish stock, W_{MSY} (cf. Fig. 1). Note that $E > E_{MSY}$ implies $W < W_{MSY}$, and vice versa.

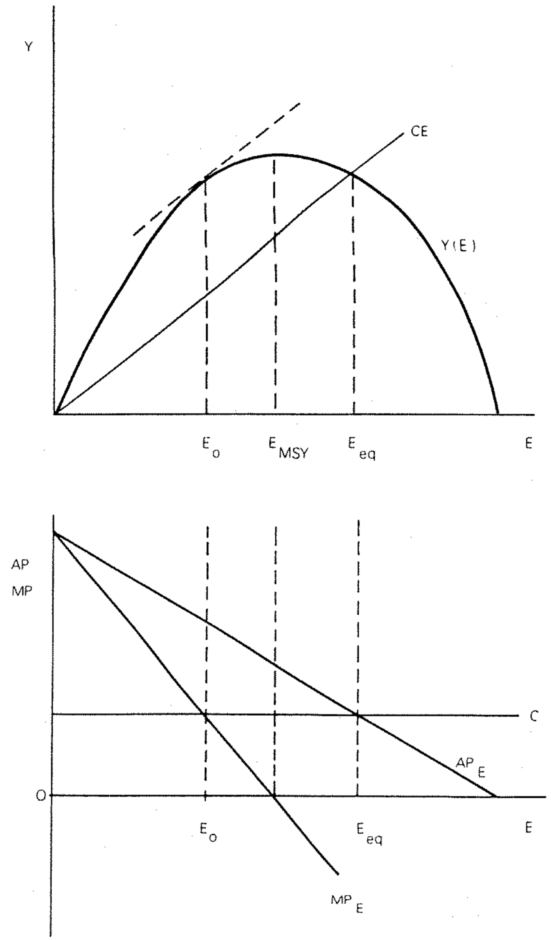

Fig. 2. Optimal (E_0), open access equilibrium (E_{eq}), and maximum sustainable yield (E_{MSY}) fishing effort.

In the lower panel of Fig. 2, the average and long-term marginal products of fishing effort are shown; that is, $AP_E = Y/E$ and $MP_E = dY/dE$, respectively. If the price of fish is independent of the amount caught, the curves showing the value of the average and marginal product will have the same shape, modified only by a change of scale on the vertical axis. Assuming, for simplicity, that the opportunity cost of fishing effort is constant, we may now compare the position yielding maximum sustainable profits with the equilibrium fishing that results if the fish stock is common property to which access is free. In the

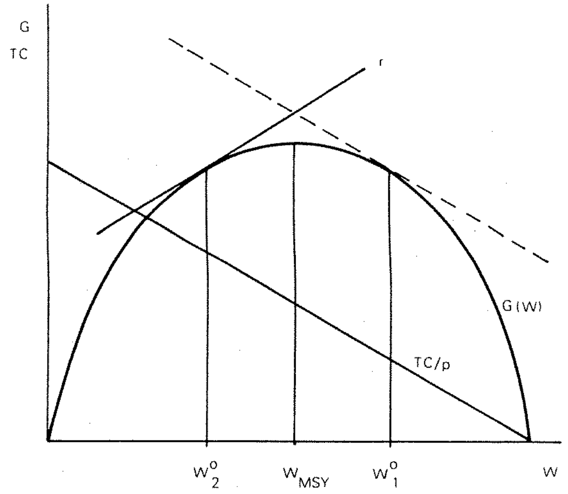

Fig. 3. Optimal stock size (W^0) when (1) sustainable rent is maximized and (2) present value of rent is maximized and the cost per unit caught is either zero or independent of stock size.

Inserting this in eqn. (6) gives

$$-G' = ch_W/(ph_E-c)$$

This expression is identical to eqn. (5) for $r=0$, as indeed it should be; sustainable rents will always prevail over transitory ones if the future is not being discounted. Adding $r>0$ on the left-hand side necessitates adjustments in E and W to preserve the equality. For some reasonable assumptions about the sign of second-order derivatives ($G''<0$, $h_{WW}\leq 0$, $h_{EE}\leq 0$, $h_{EW}=h_{WE}>0$), we are able to conclude that the necessary adjustment involves $dW<0$ and $dE>0$; that is, a dynamic analysis with a positive rate of discount shows that the optimal effort is greater and the optimal stock is less than in the static case.

One special case is particularly easy to illustrate. Let the harvest function be $h(E,W)=EqW$, as in the preceding section, and assume that the surplus growth function is $G(W)=aW-bW^2$. The biological equilibrium condition $G(W)=h(E,W)$ then determines the effort function $E(W)=(a-bW)/q$, and the total cost of fishing in a biological equilibrium becomes

$$TC = c(a-bW)/q$$

This cost function is illustrated in Fig. 3, together with the surplus growth function. Maximization of sustainable rents occurs at point W_1^0. From eqn. (5) we see that the optimum stock level would be determined by $r=G'$ if the right-hand side of eqn. (5) were zero. In that case, the optimum level of the stock would be W_2^0, which is less than W_{MSY}, a situation which a fishery biologist would be prone to call over-exploitation.

The discount rate and "investments" in fish

From Fig. 3 and eqn. (5) we see that the greater the discount rate, the lower will be the optimal level of the fish stock. However, an extremely high discount rate, that is $r\rightarrow\infty$, would be required for the economic optimum to coincide with bionomic equilibrium under free access. As explained in Section 2, the value of the short-term marginal product of fishing effort, ph_E, is equal to the opportunity cost of effort in free-access equilibrium, so that the denominator on the right-hand side of eqn. (5) approaches zero. For this to be optimal, it would be necessary that $r\rightarrow\infty$. The optimality of a free-access equilibrium may therefore be ruled out. This kind of reasoning allows us to view the free-access situation from a different angle; it does in fact "maximize" the present value of rents if future rents have no value. This is indeed an appropriate assessment from the point of view of a single fisherman or boat-owner sharing a common fish resource with many others; a fish left today to improve catches tomorrow will hardly yield any perceptible benefits to him that foregoes catching the fish in the first place.

As already noted, the optimal stock is less than that which would give maximum sustainable yield when the right-hand side of eqn. (5) is zero. This occurs either because the cost of fishing effort can be ignored ($c=0$), or because $h_W=0$. This latter case is important and not altogether unlikely, as we shall explain further in the next section. If $h_W=0$, then the size of the fish stock has no effect at all on the size of catches taken by any given level of effort. The optimum level of the stock then depends on the rate of discount and the shape of the surplus growth function only, and is independent of the price of fish, the cost of effort, and the parameters of the harvest function, except insofar as requiring that the fishery should yield positive profits.

The optimality rule $r=G'$ may be read as "the return from a marginal investment in the fish stock (G') should be equal to the rate of return on alternative investment (r)". This alerts us to the fact that eqn. (5) is simply a rule for optimum investment, requiring that the rate of return on investing in a fish stock by leaving "one unit" of fish in the sea should be equal to the rate of return on catching that unit and investing the profit in the capital market, where the rate of return is r. To see this, re-write eqn. (5) as

$$r(p-c/h_E) = G'(p-c/h_E) + ch_W/h_E$$

The left-hand side shows the return over a period of unit length of investing the profit of one unit of fish in the capital market at the going rate of interest. The profit from catching a marginal unit of fish is $p-c/h_E$, i.e. the price less the cost of that unit, where the latter is given by the cost of a unit of effort divided by the amount of fish taken by a marginal unit of effort (h_E). The right-hand side shows the returns from investing in one unit of fish in the sea over a period of unit length. The first term on the right is the increase in susto tell stories that fit this specification. Suppose that the fish are always evenly distributed in a given area. Dragging a trawl through the area will then always remove a constant proportion, q, of the fish available (W). Alternatively, suppose we put some stationary fishing gear (nets, for example) in a certain place, and that the fish move at random across the area which they inhabit and where we have put our gear. Again a constant proportion of the fish available would be caught by our gear. Both stories thus fit the specification of eqn. (7a). This specification is sometimes called "mass encounter technology", as the catch depends on how much fish the gear encounters and that, in turn, depends on the size (mass) of the fish stock.

Believable stories may also be told about the specification of eqn. (7b). In many pelagic fisheries (capelin, herring), fishermen search for suitable concentrations of fish and, having found one, shoot their gear and encircle a shoal of fish. If the size of a typical shoal of fish and the time it takes to find it are both independent of the size of the fish stock, the catch will depend on fishing effort only, so that instead of $h(E,W)$ we have $h(E)$, with eqn. (7b) as a special case. Note that this is the case discussed earlier where $h_W=0$, i.e. the size of the fish stock does not affect the catch at all, except that there must be some fish to catch ($W>0$). Since this case seems to be most likely to be encountered in fisheries that go for pelagic, schooling species, we shall refer to this as "pelagic technology".

Even if the production function characterizing any real-world fishery is most likely to be somewhere in between the polar cases of eqns. (7a) and (7b), it has some expositional merit to focus on those two. It turns out that the implications of those two for the existence of bionomic equilibrium and the risk of total depletion of stocks are quite different. This is important, since neither case need be but a slight caricature of reality.

From our discussion of the static case, it will be recalled that economic equilibrium under free access requires that the value of the average product of fishing effort be equal to the opportunity cost of effort. This implies, using eqn. (7a)

$$W_{eq} = c/pq \qquad (8)$$

That is, the opportunity cost of effort, the price of fish, and the technological constant q determine a unique level of the fish stock consistent with economic equilibrium. The biological equilibrium must lie somewhere along the surplus-growth function $G(W)$. Figure 4 shows the bionomic equilibrium at the intersection between the vertical line through W_{eq} and the surplus growth curve $G(W)$. A bionomic equilibrium with a positive and sustainable rate of exploitation will exist if W_{eq} lies between the points W_N and W_M, the natural equilibrium and the lowest viable level of the stock, respectively. From eqn. (8) we see that the location of W_{eq} is determined by the ratio c/pq. If the opportunity cost of effort is "high" or the price of fish is "low", $W_{eq} > W_N$, and it will not

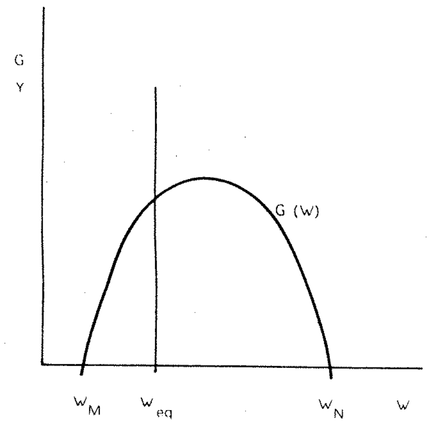

Fig. 4. Bionomic equilibrium stock (W_{eq}) with a mass-encounter technology and constant price of fish and cost per unit of effort.

be profitable to exploit the stock. If, on the other hand, c is "low" or p is "high", it is possible that $W_{eq} < W_M$. In that case, economic equilibrium will not be attained until the fish stock has been depleted below its threshold level of viability and the stock will then vanish, because of its inability to regenerate itself.

This last point alerts us to the uncomfortable fact that the only protection from extinction conferred upon common-property fish stocks is that it be unprofitable, in the short run, to deplete them below viable levels. The fact that fish stocks have rarely if ever been depleted beyond recovery is due first and foremost to the difficulty of detecting and catching "the last fish". For terrestrial animals the situation is different, and there have been cases of common-property game stocks being hunted to extinction, cf., e.g., Smith (1975).

Consider now the pelagic technology. Applying the economic-equilibrium condition that the value of the catch per unit of effort be equal to the opportunity cost of effort, and using eqn. (7b), yields

$$pq > c \qquad (9a)$$

$$pq \leq c \qquad (9b)$$

One or other of these must hold, depending on the actual values of p, c and q. Note that neither the fish stock nor fishing effort enter these equilibrium conditions at all, so that the profitability of fishing will be independent of the levels of the fish stock and fishing effort. If fishing is profitable (eqn. 9a), fishing effort will go on expanding until the fish stock has been totally depleted, and no bionomic equilibrium with a sustainable rate of exploitation will exist. If fishing is not profitable (eqn. 9b), then no fishing will take place.

This alerts us to the possibility that pelagic fish stocks that gather in large

Letting $f(Q)$ be the density function of the allowable catch-quota and using eqn. (11), noting that $f(Q)$ is time-invariant, eqn. (10) becomes

$$\max_K \sum_{t=1}^{\infty} p[\int_0^{bK} f(Q)Q\mathrm{d}Q + bK\int_{bK}^{\infty} f(Q)\mathrm{d}Q]/(1+r)^t - K$$

Solving the series and multiplying by r gives us the equivalent problem

$$\max_K p[\int_0^{bK} f(Q)Q\mathrm{d}Q + bK(1-F(bK))] - (r+a)K \qquad (12)$$

where $F(Q)$ is the cumulative distribution function of the catch-quota. Note that this expression may be interpreted as maximizing the expected value of the catch in any particular year, less the annual capacity cost. The annual capacity cost per unit of capacity is the sum of the rate of return on foregone investment (r) and the rate of capacity depreciation (a).

The first-order condition for maximum is

$$pb[1-F(bK^{\circ})] = r+a \qquad (13)$$

where K° denotes optimal capacity. The interpretation of this is straightforward. On the left-hand side we have the expected revenue from extending catch capacity at the margin; that is, the value of the marginal product of capacity (pb) times the probability that the catch-quota will be large enough to warrant full utilization of capacity. On the right-hand side we have the ex ante cost of a unit of capacity. It is clear that the higher the capacity cost ($r+a$), the less capacity is it desirable to invest in; i.e., the lower is $F(bK^{\circ})$. We may call bK° the "worthwhile quota", as this is the largest quota that it is possible to take with the optimal capacity. Larger quotas are of no value, since it costs too much in periodically idle capacity to be able to utilize them. The worthwhile quota is illustrated in Fig. 5, with arrows indicating the impact of increasing capital costs.

DO DETERMINISTIC MODELS YIELD CORRECT CAPACITY PRESCRIPTIONS FOR STOCHASTIC FISHERIES?

To make the required comparison between deterministic and stochastic models, we need a relationship between surplus growth and the size of the exploited fish stock. In the example to be discussed below, we shall use a discrete model where the stock at the beginning of period $t+1$ is related to the stock remaining after harvest in period t

$$W_{t+1} = g(W_t - h_t) \qquad (14)$$

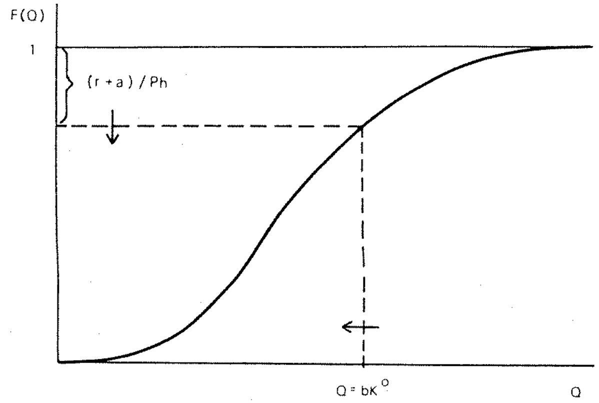

Fig. 5. Cumulative frequency distribution of catch quotas. Arrows show consequences of increasing capacity cost.

In the example to follow, we specify eqn. (14) as

$$W_{t+1} = (W_t - h_t)\exp[1-(W_t-h_t) + u_t - \sigma_u^2/2] \qquad (14')$$

This is a stochastic version of Ricker's recruitment function (cf. Ricker, 1975), where u_t is a stochastic variable, $u \sim N(0,\sigma_u^2)$, so that the distribution of W_{t+1} is lognormal and the expected value of W_{t+1} is the same as the deterministic value (i.e. with $u_t=0$, $\sigma_u^2=0$).

Figure 6 shows the function (eqn. 14'). The surplus growth is $W_{t+1}-(W_t-h_t)$, so that $G'(W_t) = \mathrm{d}W_{t+1}/\mathrm{d}W_t - 1$. In the deterministic case, this must be equal to r in the optimal solution, according to eqn. (5), for a pelagic technology. In the numerical example to follow, we shall put $r=0.1$. Calculating $\mathrm{d}W_{t+1}/\mathrm{d}W_t=1.1$ gives $W_{t+1}=W_t=0.73$ and $h_t=0.33$. As already explained, the optimal sustainable yield is independent of prices and costs, being determined simply by $G'=r$; here it is $h^{\circ}=0.33$.

As explained in the preceding section, there are two kinds of decisions to be taken in the stochastic case. First, we must decide how much of the available catch capacity should be utilized in any given situation. Second, we must decide how much capacity to invest in.

We shall separate the problem into two parts. First, we derive an optimal catch policy for a given catch capacity that is constant over time. Then we ask, what is the optimal time-invariant catch capacity, given that we follow the catch policy optimal for that capacity? The solution to this problem gives us the long-term catch capacity that is optimal on the average. The actual catch capacity will in all probability fluctuate around this average, since it would probably be desirable to postpone the replacement of worn-out equipment if there is a dearth of fish, and to speed it up if fish stocks are plentiful. How capacity, the less attractive it is to invest in capacity that is not needed for long periods, and the less attractive are large but infrequent catch quotas.

A comparison with the deterministic case is provided by the rectangular curve in Fig. 7. As we have already shown, the optimal catch capacity in the deterministic case is independent of capacity costs and equal to 0.33. However, if the present value of future catches per unit of capacity, which is equal to 10 in terms of fish at a discount rate of 10%, is less than the present value of the unit capacity cost, the fishery will not be profitable. Thus we end up with the rectangular curve in Fig. 7 as an expression of optimal catch capacity in the deterministic case. The conclusion that emerges is that uncertainty, or risk, with respect to future availability of fish clearly has an impact on the choice of optimal catch capacity, even if the fishery manager is risk-neutral. Using deterministic models as an approximation to the variability of the real world may lead us seriously astray in this regard, leading to a too large capacity if capacity costs are high, and a too small capacity if capacity costs are low. For "intermediate" unit capacity costs, i.e. those that are neither close to zero nor close to absorbing the entire resource rent, the deterministic models will give approximately correct answers, but the question remains, how much is high or low. This certainly makes a case for dealing explicitly with the variability of catch quotas.

CONCLUSION

The economic theory of fisheries started within a static and deterministic framework. Within these bounds, it was nevertheless possible to provide a clear appreciation of the fundamental economic processes, and ailments, of fisheries, even if these develop over time. In particular, it was shown how free access to common property stocks led to overexploitation.

Fisheries economics remained largely within the static framework for a long time. The explicitly dynamic analysis that became commonplace in the 1970's added refinement, showing that the static models might be misleading; in particular it showed that biological over-exploitation, and indeed extinction, could be optimal when the rate of discount is high in relation to the growth rate of the resource in question.

Explicit analysis of fish stock variability is now beginning to make its mark in the literature. An important conclusion to be drawn from this work is that variability of fish stocks caused by natural forces beyond human control is quite likely to be an important factor in determining fishing strategy and fleet capacity, even if variability, as such, is a matter of indifference. Deterministic fishery models may therefore be an inadequate guide even to long-term decisions, such as how large a fleet capacity to invest in. Practical decisions ought therefore to be based on models in which environmental variability is explicitly taken into account. This conclusion is, of course, strengthened to the extent that variability (risk) is important in its own right for the decision maker, who may, for example, prefer a predictable yield of fish farms to the uncertain yield of volatile fish stocks in the wild.

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